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Dynamical and Geometric Phases of Exciton Emission in a Semiconductor Microcavity

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Abstract By using of the invariant theory, we have studied the phase of exciton emission in a semiconductor microcavity, the dynamical and geometric phases are presented respectively. The Aharonov-Anandan phase is also obtained in the case of cyclical evolution.

Keywords Geometric phase · Exciton emission

1 Introduction

As we known that the quantum invariant theory proposed by Lewis and Riesenfeld [1] is a powerful tool for treating systems with time-dependent Hamiltonians. It was generalized in [2] by introducing the concept of basic invariants and used to study the geometric phases [3–5] in connection with the exact solutions of the corresponding time-dependent Schrödinger equations. The discovery of Berry's phase is not only a breakthrough in the older theory of quantum adiabatic approximations [3, 4], but also provides us with new insights in many physical phenomena. The concept of Berry's phase has developed in some different directions [6–14]. Recently, Jin et al. [15] have studied the collapses and revivals of exciton emission in a semiconductor microcavity. In this paper, by using of the invariant theory, we shall study the dynamical and geometric phases of exciton emission in a semiconductor microcavity.

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2 Model

The model we considered here is a microcavity containing a semiconductor quantum well embedded in a high finesse cavity. We assume that the cavity and the quantum well are ideal, and they are in an extremely low temperature situation. The quantum well interacts with cavity field via exciton, which is an electron-hole pair bound by the Coulomb interaction. The exciton and the photon modes are quantized along the direction normal to the microcavity, so we will consider the lowest-order mode in this direction. The excitons with in-plane wave vector **K** may only be dressed by the photons with the same wave vector due to the transitional invariance in the plane of the microcavity.

To further simplify the model, we will consider only one mode of photons with wave vector $\mathbf{K} = \mathbf{0}$ and frequency $\omega_c(t)$ very close to the lowest $n = 1s$ exciton energy level [16, 17]. In fact, at extremely low temperature , the thermal momentum of the excitons is so small that the thermalized excitons can be neglected [18, 19]. Combining the above considerations and neglecting the spin degrees of freedom, one can write an effective Hamiltonian for the coupled exciton-photon system as [16, 17]

$$\hat{H}(t) = \omega_c(t)\hat{a}^\dagger\hat{a} + \omega_{ex}(t)\hat{b}^\dagger\hat{b} + g(t)(\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a}) + A(t)\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b} - B(t)(\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{a} + \hat{a}^\dagger\hat{b}^\dagger\hat{b}\hat{b}), \tag{1}$$

where $\hat{b}^\dagger$ ($\hat{b}$) are creation (annihilation) operators of the excitons with frequency $\omega_{ex}(t)$, and $\hat{a}^\dagger$ ($\hat{a}$) are creation (annihilation) operators of the cavity field with frequency $\omega_c(t)$. We assume that both of them obey the bosonic commutation relation $[\hat{b}, \hat{b}^\dagger] = [\hat{a}, \hat{a}^\dagger] = 1$. The third term stands for the exciton-photon interaction with coupling strength $g(t)$, which is larger than the nonlinear interaction coefficients $A(t)$ and $B(t)$. The fourth term describes the effective exciton-exciton interaction due to Coulomb interaction. The higher-order exciton-photon interaction, the fifth term, represents the phase-space filling effects. For small in-plane wave vectors of the excitons and the photons, the nonlinear interaction constants $2A = 6Ry_{ex}a_{ex}^2/S$ and $B = g/(n_{sat}S)$, where Ry_{ex} is the binding energy of the excitons, S the quantization area, and $n_{sat} = 7/(16\pi a_{ex}^2)$ the exciton saturation density [20, 21]. The ratio of the exciton-exciton interaction constant A and the phase-space filling factor B may be determined by a degenerate four-wave mixing experiment. In this paper, we assume that these two parameters are real and positive.

3 Dynamical and Geometric Phases of Exciton Emission in a Semiconductor Microcavity

For self-consistent, we first illustrate the Lewis-Riesenfeld (L-R) invariant theory [1]. For a one-dimensional system whose Hamiltonian $\hat{H}(t)$ is time-dependent, then there exists an operator $\hat{I}(t)$ called invariant if it satisfies the equation

$$i\frac{\partial \hat{I}(t)}{\partial t} + [\hat{I}(t), \hat{H}(t)] = 0. \tag{2}$$

The eigenvalue equation of the time-dependent invariant $|\lambda_n, t\rangle$ is given

$$\hat{I}(t)|\lambda_n, t\rangle = \lambda_n|\lambda_n, t\rangle, \tag{3}$$

where $\frac{\partial \lambda_n}{\partial t} = 0$. The time-dependent Schrödinger equation for this system is

$$i\frac{\partial |\psi(t)\rangle_s}{\partial t} = \hat{H}(t)|\psi(t)\rangle_s. \tag{4}$$

According to the L-R invariant theory, the particular solution $|\lambda_n, t\rangle_s$ of (4) is different from the eigenfunction $|\lambda_n, t\rangle$ of $\hat{I}(t)$ only by a phase factor $\exp[i\delta_n(t)]$, i.e.,

$$|\lambda_n, t\rangle_s = \exp[i\delta_n(t)]|\lambda_n, t\rangle, \tag{5}$$

which shows that $|\lambda_n, t\rangle_s$ $(n = 1, 2, \ldots)$ forms a complete set of the solutions of (4). Then the general solution of the Schrödinger equation (4) can be written by

$$|\psi(t)\rangle_s = \sum_n C_n \exp[i\delta_n(t)]|\lambda_n, t\rangle, \tag{6}$$

where

$$\delta_n(t) = \int_0^t dt' \langle\lambda_n, t'| i\frac{\partial}{\partial t'} - \hat{H}(t')|\lambda_n, t'\rangle, \tag{7}$$

and $C_n = \langle\lambda_n, 0|\psi(0)\rangle_s$.

In order to obtain the exact solutions of (4), we can define operators $\hat{K}_+$, $\hat{K}_-$ and $\hat{K}_0$ as follows:

$$\hat{K}_+ = \hat{a}^\dagger\hat{b}, \qquad \hat{K}_- = \hat{b}^\dagger\hat{a}, \qquad \hat{K}_0 = \hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}, \tag{8}$$

which hold the commutation relations

$$[\hat{K}_0, \hat{K}_\pm] = \pm 2\hat{K}_\pm, \qquad [\hat{K}_+, \hat{K}_-] = \hat{K}_0, \tag{9}$$

it is easy to prove that operators $\hat{K}_+$, $\hat{K}_-$ and $\hat{K}_0$ together with the Hamiltonian $\hat{H}(t)$ construct a quasi-algebra.

Then we can get the L-R invariant as follows

$$\hat{I}(t) = \cos\theta\hat{K}_0 - e^{-i\varphi}\sin\theta\hat{K}_+ - e^{i\varphi}\sin\theta\hat{K}_-, \tag{10}$$

here θ and φ are determined by (2) and satisfy the relations

$$\dot{\theta} = 2g(t)\sin\varphi, \tag{11}$$

$$\dot{\varphi}\sin\theta\cos\varphi + \dot{\theta}\cos\theta\sin\varphi - 2g(t)\cos\theta + \sin\theta\cos\varphi[\omega_{ex}(t) - \omega_c(t) - A(t)] = 0, \tag{12}$$

$$\dot{\varphi}\sin\theta\sin\varphi - \dot{\theta}\cos\theta\cos\varphi + \sin\theta\sin\varphi[\omega_{ex}(t) - \omega_c(t) - A(t)] = 0, \tag{13}$$

where dot denotes the time derivative. In the above calculations, we have let $g(t) \gg B(t)$, $2A(t) = 3B(t)$, $\hat{N}_a = \hat{a}^\dagger\hat{a} \gg 1$, and $\hat{N}_b = \hat{b}^\dagger\hat{b} \gg 1$, and we have adopted approximations: $\langle\hat{a}\rangle = \langle\hat{a}^\dagger\rangle = \langle\hat{b}\rangle = \langle\hat{b}^\dagger\rangle$ for higher terms on the combination of operators $\hat{a}(\hat{a}^\dagger)$ and $\hat{b}(\hat{b}^\dagger)$.

According to the unitary transformation method, we can construct the unitary transformation

$$\hat{V}(t) = \exp[\sigma\hat{K}_+ - \sigma^*\hat{K}_-], \tag{14}$$

where $\sigma = \frac{\theta}{2}e^{-i\varphi}$ and $\sigma^* = \frac{\theta}{2}e^{i\varphi}$. The invariant $\hat{I}(t)$ can be transformed into a new time-independent operator $\hat{I}_V$:

$$\hat{I}_V = \hat{V}^\dagger(t)\hat{I}(t)\hat{V}(t) = \hat{K}_0. \tag{15}$$

Correspondingly, we can get the eigenvalue equation of operator $\hat{I}_V(t)$

$$\hat{I}_V|m\rangle_{\hat{a}}|n\rangle_{\hat{b}} = (m-n)|m\rangle_{\hat{a}}|n\rangle_{\hat{b}}. \tag{16}$$

In terms of the unitary transformation $\hat{V}(t)$ and the Baker-Campbell-Hausdoff formula [22]

$$\hat{V}^\dagger(t)\frac{\partial \hat{V}(t)}{\partial t} = \frac{\partial \hat{L}}{\partial t} + \frac{1}{2!}\left[\frac{\partial \hat{L}}{\partial t}, \hat{L}\right] + \frac{1}{3!}\left[\left[\frac{\partial \hat{L}}{\partial t}, \hat{L}\right], \hat{L}\right] + \frac{1}{4!}\left[\left[\left[\frac{\partial \hat{L}}{\partial t}, \hat{L}\right], \hat{L}\right], \hat{L}\right] + \cdots, \tag{17}$$

where $\hat{V}(t) = \exp[\hat{L}(t)]$, one has

$$\hat{H}_V(t) = \hat{V}^\dagger \hat{H}\hat{V} - i\hat{V}^\dagger \frac{\partial \hat{V}}{\partial t} = \alpha(t)\hat{a}^\dagger\hat{a} + \beta(t)\hat{b}^\dagger\hat{b} + \frac{\dot{\varphi}}{2}(1-\cos\theta)(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}), \tag{18}$$

where

$$\begin{aligned}\alpha(t) &= \omega_c(t)\cos^2\frac{\theta}{2} + \omega_{ex}(t)\sin^2\frac{\theta}{2} - g(t)\sin\theta\cos\varphi - \frac{3}{4}A(t)\sin^2\theta \\ &\quad - B(t)\sin\theta\cos\varphi\sin^2\frac{\theta}{2},\end{aligned} \tag{19}$$

$$\begin{aligned}\beta(t) &= \omega_c(t)\sin^2\frac{\theta}{2} + \omega_{ex}(t)\cos^2\frac{\theta}{2} + g(t)\sin\theta\cos\varphi + \frac{1}{4}A(t)\sin^2\theta - A(t)\cos^2\frac{\theta}{2} \\ &\quad + B(t)\sin\theta\cos\varphi\cos^2\frac{\theta}{2}.\end{aligned} \tag{20}$$

In the above derivations, the crossed terms $\hat{a}^\dagger\hat{b}$ and $\hat{b}^\dagger\hat{a}$ disappear in the following cases

$$[\omega_c(t) - \omega_{ex}(t) + A(t)]\sin\theta\cos\varphi + 2g(t)\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\cos 2\varphi\right) = 0, \tag{21}$$

$$[\omega_c(t) - \omega_{ex}(t) + A(t)]\sin\theta\sin\varphi - 2g(t)\sin^2\frac{\theta}{2}\sin 2\varphi = 0. \tag{22}$$

It is easy to find that $\hat{H}(t)$ differs from $\hat{I}_V$ only by a time-dependent c-number factor. Thus we can get the general solution of the time-dependent Schrödinger equation (4)

$$|\Psi(t)\rangle_s = \sum_n\sum_m C_{nm}\exp[i\delta_{nm}(t)]\hat{V}(t)|m\rangle_{\hat{a}}|n\rangle_{\hat{b}}, \tag{23}$$

with the coefficients $C_{nm} = \langle n, m, t=0|\Psi(0)\rangle_s$. The phase $\delta_{nm}(t) = \delta^d_{nm}(t) + \delta^g_{nm}(t)$ includes the dynamical phase

$$\delta^d_{nm}(t) = -m\int_{t_0}^t \alpha(t')dt' - n\int_{t_0}^t \beta(t')dt', \tag{24}$$

where $\alpha(t')$ and $\beta(t')$ have been given in (19) and (20), and the geometric phase

$$\delta^g_{nm}(t) = \int_{t_0}^t (n-m)\frac{\dot{\varphi}}{2}(1-\cos\theta)dt'. \tag{25}$$

Particularly, the geometric phase becomes in the case of considering the cyclical evolution

$$\delta_{nm}^{g}(t) = \frac{1}{2}\oint (n-m)(1-\cos\theta)d\varphi, \tag{26}$$

which is the known geometric Aharonov-Anandan phase.

4 Conclusions

In conclusion, we have studied the phase of exciton emission in a semiconductor microcavity by using of the L-R invariant theory, the dynamical and geometric phases are presented, respectively. In particular, the Aharonov-Anandan phase appears when we consider the cyclical evolution.

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